

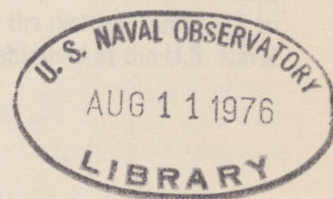
INTRODUCTION

U N I T E D S T A T E S N A V A L O B S E R V A T O R Y

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June 1, 1976



THE DETERMINATION OF UNIVERSAL TIME AT

THE U. S. NAVAL OBSERVATORY

by

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INTRODUCTION

Universal time (UT) is the form of mean solar time most widely used as the basis for timekeeping in the world today. The natural period of UT is a fraction of the actual rotational period of the Earth with respect to a frame of reference fixed in space. In practice, it is determined from the astronomical observations of stars. This observed time is designated UTO. Corrections for the changing orientation of the rotational pole with respect to the surface of the Earth are applied to UTO to obtain the time scale UT1. The rotational speed of the Earth is known to vary seasonally, for which corrections empirically determined from past observations are applied to UT1 to arrive at the time scale UT2. The basis for most civil timekeeping, however, is the UTC time scale which is a clock time not strictly defined in terms of the rotation of the Earth. It is regularly adjusted by a leap second if necessary so that it does not differ from UT1 or UT2 by more than a specified amount of time. (See Appendix A).

Astronomical observations used in the determination of UT are made by the U.S. Naval Observatory using photographic zenith tubes (PZT) located in Washington, D.C., and Richmond, FL. This instrument has been described by Markowitz (1960). The purpose of this paper is to review the relationship of UT to the rotation of the Earth and to outline the way in which the system of UT is obtained at the U.S. Naval Observatory.

UNIVERSAL TIME AND THE ROTATION OF THE EARTH

The rotational velocity of the Earth is known to experience small changes so that any time scale based on the Earth's rotational period is necessarily non-uniform. Atomic time is defined in terms of the frequency of an atomic energy level transition occurring in cesium atoms and thus provides a time scale which is not related to the rotation of the Earth and is presumably uniform. The U.S. Naval Observatory atomic time scale is designated A.1 (see Appendix A). Comparison of UT with a uniform time scale yields information concerning the actual rotational speed of the Earth.

Universal time is defined as "12 hours + the Greenwich hour angle of a point on the equator whose right ascension, measured from the mean equinox of date is: $R_u = 18^h 38^m 45^s.836 + 8640184^s.542 T_u + 0^s.0929 T_u^2$, where T_u is the number of Julian centuries of 36525 days of universal time elapsed since the epoch of Greenwich mean noon (regarded as 12^h UT) on 1900 January 0." (Explanatory Supplement, p.73). This point is not identified with any physically observable point. It is purely a point used to define UT.

In practice, UTO is determined from the astronomical observations of the diurnal motion of stars through the intermediary of sidereal time. The sidereal time at any instant at any location is defined as the local hour angle of the equinox at that instant. Since the equinox is not an observable point, stars are observed whose positions with respect to the instantaneous equinox are well known.

An observation of a star in this case usually is the noting of the time of the transit of a star across the observer's meridian on some reference clock. This observation may be affected by accidental and systematic errors in the assumed position of the star, refraction in the atmosphere and by variations in the observer's meridian produced by motion of the rotational pole with respect to the observer and by lunisolar tidal influences on the direction of the vertical. With the PZT the UTO time of transit of a star across the meridian of the instrument is computed using the expression

$$T_c = (\alpha + \lambda - ST - EQ)r, \quad (1)$$

where

- α = apparent right ascension of the star,
- λ = astronomical longitude of the instrument,
- ST = Greenwich mean sidereal time at 0^h UT,
- EQ = equation of the equinoxes, and
- r = ratio of mean solar to mean sidereal time interval
(0.997269566414 at epoch 12^h UT January 0, 1900).

The apparent right ascension is computed using a standard method allowing for the effects of precession, nutation, stellar and diurnal aberration (Explanatory Supplement, p. 145; Woolard and Clemence, p. 299). The longitude adopted for Washington PZT #3 is $5^h 8^m 15^s.7094$. That for the Richmond PZT #2 is $5^h 21^m 31^s.7030$. The Greenwich mean sidereal time at 0^h UT is computed from the expression

$$ST = 6^h 38^m 45.836 + 8640184.542 T_u + 0.0929 T_u^2 \quad (2)$$

(Explanatory Supplement, p.75). The equation of the equinoxes is the difference between apparent and mean sidereal time and is computed from the theory of nutation (Explanatory Supplement p.43).

The part of (1) within the parentheses represents twelve hours of mean sidereal time + the interval in mean sidereal time between the passage of the star and the point R_u across the local meridian. This sidereal interval is then converted to a universal time interval using the ratio of mean solar to mean sidereal time interval.

Four exposures of the star are made with the PZT as the star crosses the instrumental meridian. The mean epoch in UTC of the four exposures can be denoted as T_0 . The hour angle of the star at T_0 is found

from the reduction of the measurements of the star's images on the photographic plate and is designated H_0 . The difference $T_c - (T_0 - H_0)$ is then the quantity $UTO - UTC$ as determined from the observation of that star.

Since the instantaneous pole of rotation is moving with respect to the surface of the Earth the instrumental meridian is subject to a small variation (Woolard, 1959; Munk & MacDonald, 1960; Woolard and Clemence, p. 202; Mueller, 1969). The positions of the rotational pole (x,y) with respect to the Conventional International Origin (CIO) can be used to correct for this effect (see Appendix B). This correction of UTO to UTI is accomplished by applying the expression

$$UTI = UTO + (x \sin \lambda - y \cos \lambda) \tan \phi / 15,$$

where ϕ is the adopted astronomical latitude of the instrument. For Washington PZT #3, $\phi = 38^\circ 55' 17''.243$ and for Richmond PZT #2, $\phi = 25^\circ 36' 47''.157$.

In addition, corrections for annual and semi-annual variation in the rotational speed of the Earth due to meteorological and tidal effects are applied to obtain UT2. The empirically determined expression which has been in use as recommended by the BIH since March 1962 is

$$UT2 = UT1 + 0^s.022 \sin 2\pi T - 0^s.012 \cos 2\pi T \\ - 0^s.006 \sin 4\pi T + 0^s.007 \cos 4\pi T, \quad (3)$$

where T represents the fraction of the Besselian year of the observation. Before this date the expression used was

$$UT2 = UT1 + 0^s.022 \sin 2\pi T - 0^s.017 \cos 2\pi T \\ - 0^s.007 \sin 4\pi T + 0^s.006 \cos 4\pi T. \quad (4)$$

The mean sidereal day is equal to the length of time between two successive transits of the mean vernal equinox through the same meridian. Because of the effect of precession on the mean equinox, this length of time is not equal to the rotational period of the Earth with respect to a reference frame fixed in space. Using the presently adopted value for the constant of precession, the sidereal day is found to be shorter than the period of rotation by $0^s.008412 + 5^s.086 \times 10^{-6} T_u$ (Woolard and Clemence, p.351). The ratio between the mean solar (UT) day and the rotational period of the Earth is 1.002737811906 (Explanatory Supplement, p. 76).

If we let the rotational speed of the Earth as a function of time (t) be given by $\Omega(t) = 2\pi/T_d$ where T_d is the length of the UT day and if $\Omega_0 = 2\pi/86400$ seconds of atomic time (A.1), then $\Omega(t) = \Omega_0 - \omega(t)$. The universal time at an epoch T is then given by

$$UT = \int_{T_0}^T \Omega(t) dt, \text{ where } t \text{ is in atomic time.}$$

$$\text{Then } UT = \int_{T_0}^T \Omega_0 dt - \int_{T_0}^T \omega(t) dt, \text{ or}$$

$$A.1 - UT = \int_{T_0}^T \omega(t) dt.$$

$$\text{Thus } d/dt(A.1 - UT) = \omega(t). \quad (5)$$

DETERMINATION OF THE ADOPTED A.1 - UT1

Values of the quantity $A.1 - UT1$ are obtained from observations made with the PZT's located in Washington and Richmond. $UT1 - UTC$ is determined for each night on which observations were made. These are combined with values of $A.1 - UTC$ (see Appendix A) to obtain $A.1 - UT1$ for each station. These are combined to produce an adopted curve of $A.1 - UT1$ which has been published in the U.S. Naval

Observatory Time Service Publications Series II.

The method used to determine this adopted A.1 — UT1 curve must consider that observations are not available on all nights of the year and that each night's value is the mean of a varying number of individual star observations. In addition, systematic differences of a quasi-periodic nature exist between the data of two instruments. This is largely due to the systematic errors in the catalogs of star positions and proper motions that are used in the reduction of the observations and to the different environmental conditions that exist at the two sites.

Until 1 January 1974 the results of the two stations were combined visually. Each night's value was plotted and a smooth curve was drawn through the data for each station. The resulting two curves were then combined using a weighted mean for each day. The Richmond curve received a weight of two while that of Washington received a weight of unity.

Beginning on 1 January 1974 the A.1 — UT1 data published in the Series II publication are the result of a numerical smoothing technique using the method of Vondrak (1969). This technique is based on the minimization over the interval r through s containing N points of the quantity $F + \lambda^2 S$, where

$$F = \sum_{i=1}^N P_i (y_i - y_{\text{obs } i})^2 ,$$

P_i = weight of i^{th} observation ,

y_i = value of the curve $\Phi(x)$ fit through the data,

$y_{\text{obs } i}$ = observed value of A.1 — UT1,

$$S = \int_r^s \left[\frac{d^3}{dx^3} \Phi(x) \right]^2 dx ,$$

λ^2 = constant used to regulate the degree of smoothness.

The usual least-squares condition is obtained with the minimization of F alone. The constant λ^2 must be chosen empirically. As pointed out by Vondrak, if λ^2 is zero, the solution is $y_i = y_{\text{obs } i}$, and the curve becomes "rougher". Similarly as λ^2 becomes very large, S must approach zero and the curve becomes "smoother". A Lagrange polynomial of the third order is used for $\Phi(x)$. The observed values in this case are residuals from a least-squares parabola which is fitted to the raw data (A.1 — UT2) over the interval of N sights. Each residual representing one night's value of A.1 — UT2 is weighted by the number of stars used in the determination.

Vondrak's method is well suited for this application since equal data intervals are not required, the observations may be weighted, and the degree of roughness of the resulting curve may be regulated. In this case, Vondrak's method is applied to the data of both stations to produce two separate curves. An interval of 21 points ($N = 21$) is treated at one time. This represents 21 nights on which observations were obtained and may not represent 21 consecutive nights. The Vondrak method provides a fit of the 21 points, but only the value of the curve for the eleventh point (the mid-point of the 21 point interval) is used as the fit value, $\Phi(x)$, of A.1 — UT2 for that night and instrument. The interval is then advanced by one data point (not necessarily one night) by deleting the earliest point and adding the next consecutive data point. This "running" process is repeated until a smoothed value of A.1 — UT2 is produced for every data point with the exception of the earliest and latest ten points contained in the data set. The value of λ^2 was determined empirically using past observations and the previously adopted curve of A.1 — UT2. The adopted value in this case is $\lambda^2 = 10^6$.

This procedure has been adopted because each observation is treated alike mathematically. It also allows later observations to be added to the data set and treated the same as past data without any loss of continuity. There are then two sets of smoothed data, $\Phi(x)$, representing the smoothed observations of

the Washington and Richmond PZT's. To complete the sets for those dates on which no observations were possible, an interpolation scheme is used. A Lagrange polynomial of the sixth order is used to interpolate the value of $A.1 - UT2$ for each date on which no observations were obtained. The three points on each side of the missing value are used in this procedure. This technique was arrived at empirically by using past observations and the previously adopted values of $A.1 - UT2$. A complete set of smoothed $A.1 - UT2$ is thus obtained for each day for each PZT.

The two curves are then combined to produce the curve of the adopted $A.1 - UT2$. Judging from past results, no particular weighting scheme appears to be justified in the combination of the two curves. Thus, the two curves are combined by integrating the average of their first derivatives which is an estimate of $\omega(T)$ as given in (5). The first derivative is found for each day, T , for both Washington and Richmond. These are averaged to produce the quantity $\omega(T)$ and the value of $A.1 - UT2(T)$ is then given by

$$A.1 - UT2(T) = A.1 - UT2(T - 1^d) + \omega(T) \times 1^d.$$

This procedure was adopted because, while there may be systematic differences in the Richmond and Washington curves of $A.1 - UT2$ of up to $\pm 0^s.020$, the daily variation of both curves does not show large differences. The curve is adjusted so that the value of $A.1 - UT2$ is equal to the value of the Richmond and Washington curves when they are equal. Finally, the difference, $UT2 - UT1$, is applied to all values of the curve to obtain the adopted curve of $A.1 - UT1$.

The PZT observations made since 1 June 1955 have been treated using the Vondrak procedure. The graphical method has been used for data obtained from 1 January 1956 through 31 December 1973, and these values were originally published in Time Service Publication Series 11 as they became available. Since the date of publication these data have been adjusted for the redetermined polar coordinates and longitude change. The two sets of adopted $A.1 - UT1$ are compared in Figure 1. Differences are seen which are most likely due to the fact that the recent treatment using Vondrak's procedure made use of observations which were re-reduced using an improved catalog of star positions and proper motions (McCarthy, 1973). The previously published values are based on data reduced using the unimproved catalog for Washington through 1969 and the unimproved catalog for Richmond through 1971. Also, different formulae were used for $UT2 - UT1$ before March 1962. The data set produced graphically made use of (4) before 1962 while the recent set makes use of (3) for all of the data since 1955. The annual periodicity with increasing amplitude apparent in Figure 1 is due to the differences in the proper motion systems between the improved and unimproved catalogs. When both sets of data make use of the improved catalogs beginning in 1972, the annual term disappears. Generally, the differences between the two sets of adopted values of $A.1 - UT1$ are less than $\pm 0^s.005$ but differences of up to $0^s.018$ are seen.

The data used in the determination of the adopted $A.1 - UT1$ along with the values of the adopted curve as obtained using the Vondrak procedure are available from the U.S. Naval Observatory in Time Service Publication Series 6. The raw data have been reduced using one consistent reduction procedure and the improved catalog. This data set supersedes all data previously published in the Series 11 publications and should be the only set used for analysis. Current observations and adopted curves of $A.1 - UT1$ obtained using the Vondrak procedure will continue to be published in future Series 11 publications.

APPENDIX A A.1 & UTC TIME SCALES

The U.S. Naval Observatory system of atomic time, A.1, which exists for dates since 1 June 1955 is based on the frequency obtained in an energy level transition occurring in cesium atoms (Essen, et al., 1958; Markowitz, et al., 1958). It is defined as a system of time in which the second is equal to the length of time which is required for "9 192 631 770 oscillations of cesium at zero field" (Time Service Notice No. 6). The epoch of A.1 was defined so that on 1 January 1958 at 0^h0^m0^s UT2 the value of A.1 was also equal to 0^h0^m0^s. The epoch 0^h0^m0^s UT2 refers to the system of UT2 which was in use at that time.

Observations made with the cesium standards of Essen and the Naval Research Laboratory were used to compute the early portion of the atomic time scale. As more cesium clocks came into use around the world, the U.S. Naval Observatory made use of their atomic frequencies to obtain A.1. This was done through the frequency monitoring of VLF stations by the laboratories maintaining the cesium clocks. The frequencies reported were intercompared to obtain a mean atomic frequency. Beginning in 1967 the A.1 time scale was determined solely from a number of cesium clocks located at Washington and at Richmond (Winkler, et al., 1970). This time scale is the same one which has been designated as AT (USNO) by the Bureau International de l'Heure (BIH). A.1 differed from the International Atomic Time (TAI) maintained by the BIH by 0^s.0343985 on 3 January 1972 (BIH Circular D65, Section 4), and may have a rate with respect to TAI which should not exceed ± 5 parts in 10^{13} .

The source of UTC used for astronomical observations by the U.S. Naval Observatory until 20 September 1967 was the time of emission of WWV, since this station could be received at both Washington and Richmond. This time scale is equivalent to UT2C as it was sometimes referred to in Time Service Publications Series II. From 20 September 1967 until 1 January 1972, the source of UTC for the U.S. Naval Observatory was the cesium clock array at the Observatory with an offset in frequency which was determined by the BIH. The offset was determined so that UTC would closely follow the UT2 time scale (BIH Annual Reports, 1968 – 1972). In addition, step adjustments in the epoch of UTC were made occasionally by the BIH so that $|UT2 - UTC| < 0^s.1$.

Beginning on 1 January 1972 the definition of UTC was changed to comply with Recommendation 460 of the International Radio Consultative Committee (CCIR) so that the frequency of a UTC clock is now the atomic frequency of cesium without an offset. On 1 January 1972 UTC was adjusted so that the difference $TAI - UTC = 10^s.0000000$ (BIH Annual Report, 1973). The epoch of UTC is now adjusted by one second when necessary, preferably on either the last day of December or June, so that $|UT1 - UTC| < 0^s.8$. These changes are announced by the BIH.

APPENDIX B

POLAR COORDINATES USED TO DETERMINE UT1 – UTO

The x,y coordinates of the pole of rotation with respect to the CIO used by the U.S. Naval Observatory are obtained from the Bureau International de l'Heure (BIH). For the dates from 1 June 1955 to 31 December 1961 the coordinates were obtained from the Circulars B/C published by the BIH plus corrections found in the BIH Annual Report for 1968 (p. 37). The coordinates for the period 1 January 1962 through 4 January 1968 were obtained from the BIH Annual Report for 1970 (pp. 53 – 62). From that time until 9 September 1974 polar coordinates were obtained from BIH Circulars B/C. Currently this information is obtained from the monthly BIH Circular D.

Murray, D.D., 1973, *Astron. J.*, 78, 642.

Mueller, U., 1969, *Spherical Astronomy*, 2nd ed., Springer-Verlag, New York.

Munk, W.R. and G.J.F. MacDonald, 1950, *The Rotation of the Earth*, Cambridge University Press, London.

USNO, *General Notice No. 6*, 1959, U.S. Naval Observatory, Washington.

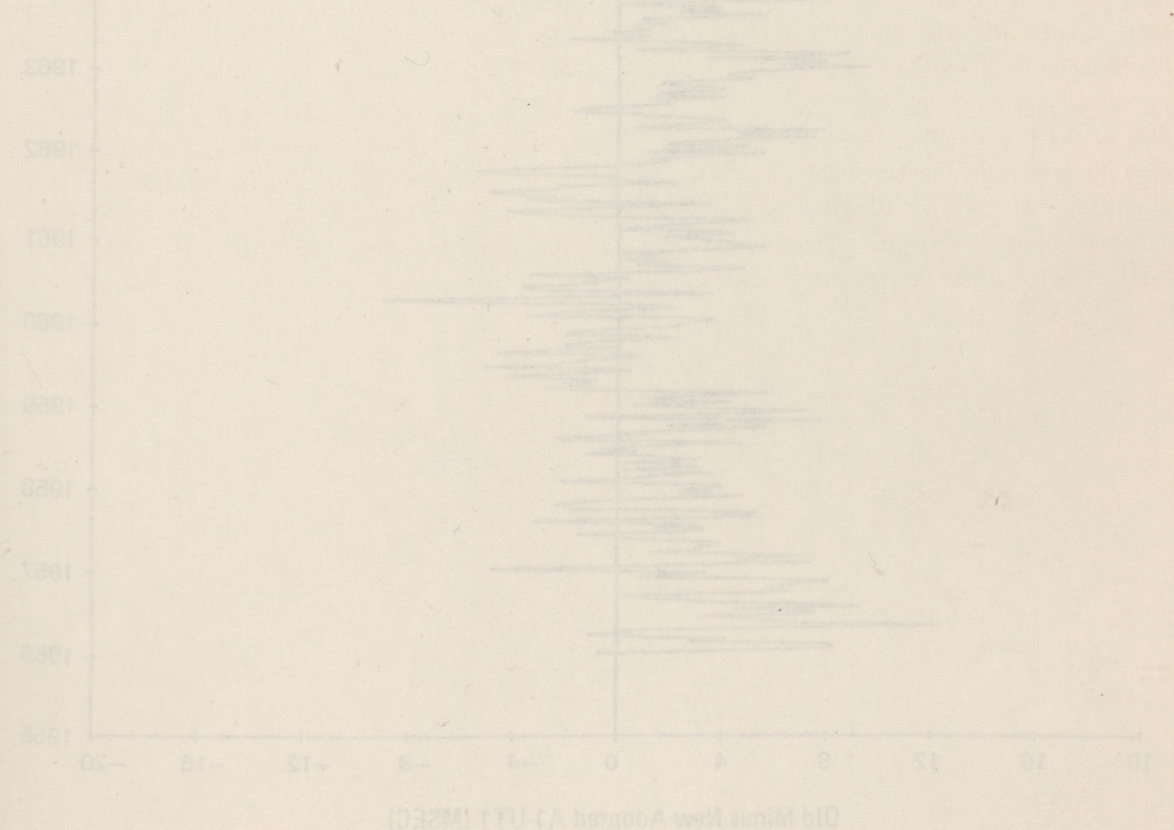
Vondra, J., 1968, *Bull. Astron. Inst. of Czechoslovakia*, 15, 1.

Wahr, J.M., R.B. Hall, and D.B. Percival, 1975, *Mon. Not. R. Astr. Soc.*, 125, 1.

Woolard, E.W., 1959, *Astron. Papers Prepared by Members of the American Philosophical and Naturalists Association*, XV, Part 1.

Woolard, E.W. and G.M. Clemencia, 1966, *Spherical Astronomy*, Academic Press, New York.

USNO, *Observatory Time Service Publications*, Series A, No. 1.



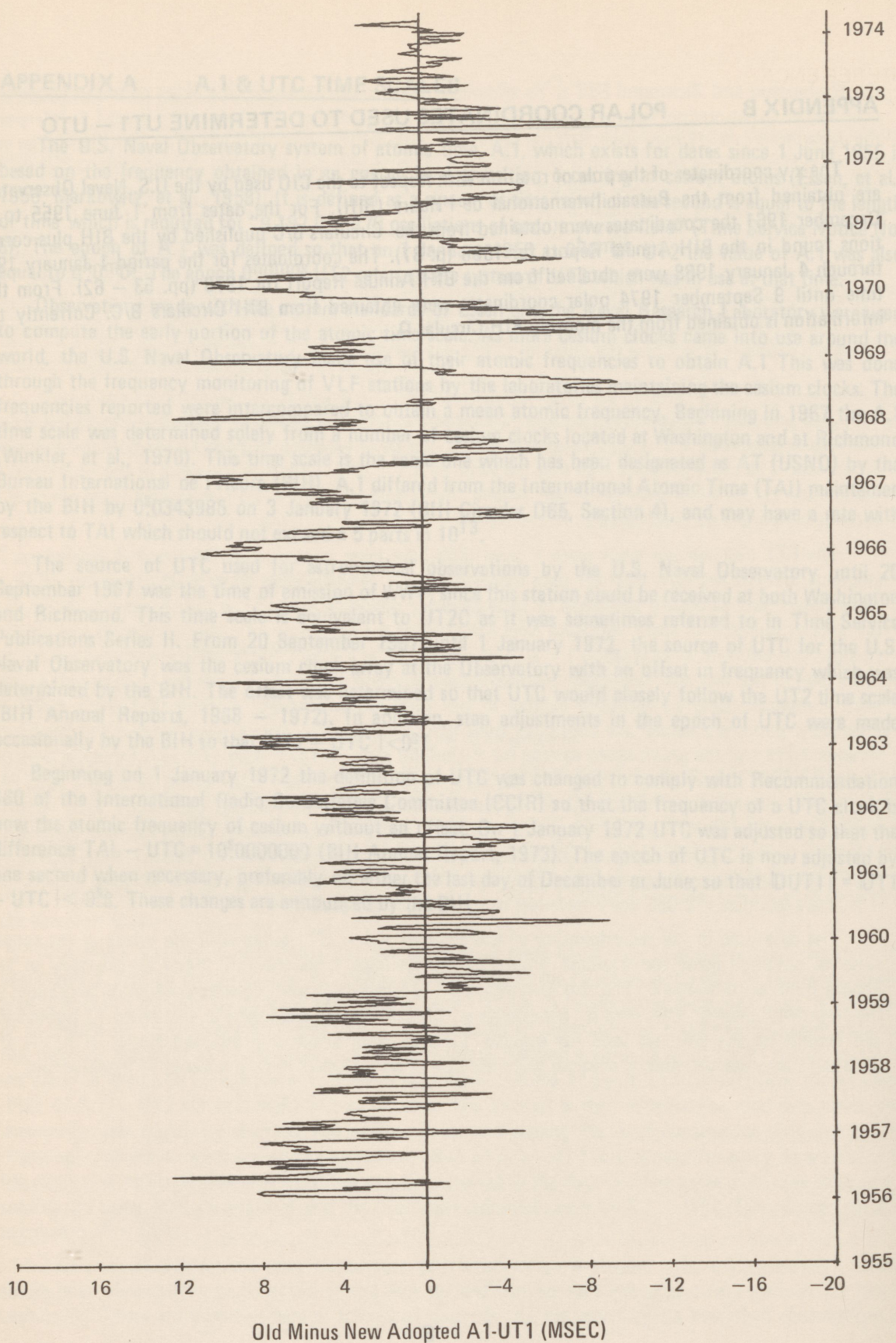


Figure 1. Comparison of previously published values of adopted A.1-UT1 (old) and the redetermined values (new).

REFERENCES

- Annual Report of the Bureau International de l'Heure*, 1968-1973. Bureau International de l'Heure, Paris.
- Essen, L., J.V.L. Parry, W. Markowitz, and R.G. Hall, 1958. *Nature* **181**, 1954.
- Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac*, 1961. Her Majesty's Stationery Office, London.
- Markowitz, W., 1960. "The Photographic Zenith Tube and the Dual-Rate Moon Position Camera", in *Telescopes*, G.P. Kuiper and B.M. Middlehurst, editors, University of Chicago Press, Chicago.
- Markowitz, W., R.G. Hall, L. Essen, and J.V.L. Parry, 1958. *Phys. Rev. Letters*, **1**, 105.
- McCarthy, D.D., 1973. *Astron. J.*, **78**, 642.
- Mueller, I.I., 1969. *Spherical and Practical Astronomy as Applied to Geodesy*, Frederick Ungar Publishing Co., New York.
- Munk, W.H. and G.J.F. MacDonald, 1960. *The Rotation of the Earth*, Cambridge University Press, London.
- Time Service Notice No. 6*, 1959. U.S. Naval Observatory, Washington, D.C.
- Vondrak, J., 1969. *Bull. Astron. Inst. of Czechoslovakia*, **20**, 349.
- Winkler G.M.R., R.G. Hall, and D.B. Percival, 1970. *Metrologia*, **6**, 126.
- Woolard, E.W., 1959. *Astron. Papers Prepared for the Use of the American Ephemeris and Nautical Almanac*, **XV**, Part 1.
- Woolard, E.W. and G.M. Clemence, 1966. *Spherical Astronomy*, Academic Press, New York.
- U.S. Naval Observatory Time Service Publications Series 6, No. 3.